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## Modeling time-varying uncertain situations using Dynamic Influence Nets <sup>☆</sup>

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## ABSTRACT

This paper enhances the *Timed Influence Nets* (TIN) based formalism to model uncertainty in dynamic situations. The enhancements enable a system modeler to specify persistence and time-varying influences in a dynamic situation that the existing TIN fails to capture. The new class of models is named *Dynamic Influence Nets* (DIN). Both TIN and DIN provide an alternative easy-to-read and compact representation to several time-based probabilistic reasoning paradigms including Dynamic Bayesian Networks. The Influence Net (IN) based approach has its origin in the Discrete Event Systems modeling. The time delays on arcs and nodes represent the communication and processing delays, respectively, while the changes in the probability of an event at different time instants capture the uncertainty associated with the occurrence of the event over a period of time.

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### 1. Introduction

Bayesian Networks (BN) [16,22,23] have been extensively used in the last two decades for modeling and reasoning in a variety of uncertain domains including medical diagnosis, human belief systems, forecasting, sensor fusion, system troubleshooting, etc. A BN is a graphical representation of probability distributions. It consists of two components. The first is a directed acyclic graph in which each node represents a random variable, while the set of arcs connecting pairs of nodes represents certain conditional independence properties. This component captures the *structure* of the probability distribution. The second component is a collection of *parameters* that describe the conditional probability of each variable given its parent in the graph. Together, these two components represent a unique probability distribution [23].

BN were originally designed to capture *static* interdependencies among variables in an uncertain situation. The last few years have seen an emergence of techniques that attempt to integrate the notion of time and uncertainty. The most popular of them is called Dynamic Bayesian Network (DBN) [19]. A DBN is created by discretizing time and creating instances of variables in a BN for each point in the time interval under consideration. Having its roots in canonical Bayesian Network, DBN suffers from the same limitation as BN, i.e., (a) intractability of inference and (b) elicitation of all the conditional probabilities. The first issue deals with computing the likelihood of variables of interest in reasonable amount of time, while the second issue deals with modeling complex situations using minimum amount of information. Several attempts have been made to address both issues. Approximate and simulation-based algorithms have been proposed that exploit certain conditions in a DBN to efficiently compute the likelihood of variables of interest in a reasonable amount of time [3,7,17,20,26]. Efforts have also been made to add different types of temporal constructs to the existing BN formalism to model various types of dynamic

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situation. Hanks and Madigan [14] provides an excellent and elaborative overview of different knowledge elicitation techniques to time-dependent probabilistic reasoning. A brief account of some of the important contributions is presented below.

The work of Santos and Young [25] focuses on using Allen's interval logic [2] for knowledge elicitation, while Burns and Morrison [4] have proposed a template, based on Allen's interval logic, for structured temporal reasoning. Figueroa and Sucar [9] proposed Temporal Bayesian Network of Events (TBNE) in which each temporal node represents an event or a state change of a variable. Time is discretized in a finite number of intervals and each interval for a child node represents possible delays between the occurrence of one of its parent event and the corresponding child state change. This, however, poses a limitation in cases where evidence about an event can be explained by several causes. Galan and Diez [10] proposed Networks of Probabilistic Events in Discrete Time (NPEDT). Their approach discretizes time and each state of a variable represents the time instants at which the event occurs. Thus, the situations where a variable changes state at multiple time instants cannot be easily modeled with the approach. They have also suggested several types of temporal noisy gates which are extensions of canonical noisy gates in BN. Galan et al. [11] have compared the performances of TBNE and NPEDT for temporal fault diagnosis in an industrial domain and have reported that NPEDT performs better than TBNE for a particular class of problems. Tawfik and Neufeld [27] have proposed Temporal Bayesian Networks. Their approach models CPTs as a function of time but requires a very detailed understanding of the underlying dynamic process. Hanks et al. [15] have proposed a simulation-based semi-Markov model for dynamic processes where the state of a system changed due to both exogenous and endogenous events.

Wagenhals et al. [32] have added several temporal constructs to Influence Nets [24], a special instance of Bayesian Networks, that allow a system modeler to specify communication and information processing delays associated with nodes and arcs in the network. The constructs also allow a system analyst to associate a time stamp with an event. The time stamp represents the time at which the corresponding action is executed. The class of model is referred to as Timed Influence Nets (TINs). A TIN allows a system analyst to observe the changes in the probability of a particular event over a period of time. TINs have been experimentally used in the area of Effects-Based Operations (EBOs) for evaluating alternate courses of action and their effectiveness to mission objectives [29,30,33,34]. Haider and Zaidi [12] have developed an algorithm that transforms a TIN into a DBN. They suggest TIN for knowledge elicitation and courses of action evaluation and DBN for incorporating the evidence that arrives during the execution of the course of action. Haider et al. [13] have integrated point interval temporal logic [36], an extension of Allen's interval logic, into TIN-based formalism to answer temporal queries and to perform what-if analyses.

Despite their ability to model complex situations in a compact and easy to read manner, TINs fail to capture certain types of dynamic situations. For instance, they do not have the capability to model the impact of different sequences of actions. Thus, no matter what the sequence of action is, the final outcome remains the same. Furthermore, they assume that the influence of an event on another event is stationary, i.e., the influence remains the same throughout the campaign. Both of these constraints may turn out to be unrealistic in many real world situations. The paper proposes structural and parametric enhancements to TIN to overcome the above limitations. The structural enhancement would enable a system analyst to model the impacts of different sequences of actions on the desired effect; while the parametric enhancements would aid the mathematical modeling of time-varying influences. Together these enhancements make it possible to model the impact of repetitive actions in a dynamic uncertain situation.

The rest of the paper is organized as follows. Section 2 describes Influence Nets and its knowledge elicitation technique, namely CAST logic. Timed Influence Nets is described in Section 3. The limitations of the TIN-based formalism are explained in Section 4 along with ways of overcoming those limitations. The formal description of the new class of models, named Dynamic Influence Nets, is explained in Section 5 along with some examples. Finally, Section 6 provides conclusion and the future research directions.

## 2. Influence Nets

Influence Nets are Directed Acyclic Graphs (DAGs) where nodes in the graph represent random variables, while the edges between pairs of variables represent causal relationships. The modeling of the causal relationships is accomplished by creating a series of cause and effect relationships between variables representing desired effect(s) and variables representing set of actionable events. The actionable events are drawn as root nodes (nodes without incoming edges), while the desired effect is modeled as a leaf node (node without outgoing edges). Typically, the root nodes are drawn as rectangles while the non-root nodes are drawn as rounded rectangles. Influence Nets require a system modeler (or a subject matter expert) to specify the CAST logic parameters instead of the probabilities. The required probabilities are internally generated by the CAST logic algorithm with the help of user-defined parameters. The following items characterize an IN while a formal definition is given in Definition 1.

1. A set of random variables that makes up the nodes of an IN. All the variables in the IN have binary states.
2. A set of directed links that connect pairs of nodes.
3. Each link has associated with it a pair of CAST logic parameters that shows the causal strength of the link (usually denoted as  $h$  and  $g$  values).
4. Each non-root node has an associated CAST logic parameter (denoted as the baseline probability), while a prior probability is associated with each root node.

**Definition 1**

An Influence Net is a four-tuple **(V, E, C, B)** where

- V**: set of Nodes,
- E**: set of Edges,
- C** represents causal strengths:  
 $E \rightarrow \{(\mathbf{h}, \mathbf{g}) \text{ such that } -1 < \mathbf{h}, \mathbf{g} < 1\}$ ,
- B** represents a Baseline or Prior probability:  
 $V \rightarrow [0,1]$

Fig. 1 shows an example of an Influence Net. Nodes B and E represent the actionable events (root nodes) while node D represents the desired effect (leaf node). The directed edge with an arrowhead between two nodes shows the parent node promoting the chances of a child node being true, while the roundhead edge shows the parent node inhibiting the chances of a child node being true. The text associated with the non-root nodes represent the corresponding conditional probability values obtained from the CAST logic parameters (not shown in the figure) while the text associated with the root nodes represents the prior probabilities. The values associated with arcs show time delays and are discussed in Section 3. The probability propagation in an IN is based on the “independence of parents” assumptions (similar to the loopy belief propagation [8,18,21,23]) Thus, the marginal probability of a non-root node is computed with the help of its conditional probability table (CPT) and the prior probabilities of its parents. For instance, the marginal probability of variable A is computed as

$$P(A) = P(A | -B, -E)P(-B)P(-E) + P(A | -B, E)P(-B)P(E) + P(A | B, -E)P(B)P(-E) + P(A | B, E)P(B)P(E) = 0.06 \quad (1)$$

The probability of D is then computed by using its CPT and the marginal probabilities of A (computed above) and E.

$$P(D) = P(D | -E, -A)P(-E)P(-A) + P(D | -E, A)P(-E)P(A) + P(D | E, -A)P(E)P(-A) + P(D | E, A)P(E)P(A) = 0.11 \quad (2)$$

In this way, marginal probabilities are propagated in the forward direction, i.e., from the root nodes to the leaf nodes.

**2.1. CAST logic**

The specification of a Bayesian Network requires an exponential number of parameters for model specification. As a model grows larger, this requirement presents a very big challenge to a system modeler. As an attempt to overcome this limitation, Chang et al. [5] developed a formalism called CAusal STrength (CAST) logic to elicit the large number of conditional probabilities from a small set of user-defined parameters. The logic has its roots in the Noisy-Or approach [1,8]. In fact, it can be shown that the Noisy-Or approach is a special case of the CAST logic. The logic requires only a pair of parameter values for each dependency relationship between any two random variables. The values are converted into conditional probability tables and the resultant tables are used during the probability propagation phase. Thus, Influence Nets could be regarded as a special instance of Bayesian Networks. A brief explanation of the CAST logic is provided below with the help of an example shown in Fig. 2. Readers interested in a detailed description of the CAST logic should refer to [5,24].

Fig. 2 contains four nodes A, B, C and X. On each arc, two causal strengths are specified. These numbers represent the probability that a specified state of a parent node will cause a certain state in the child node. Positive values on arcs are causal influences that cause a node to occur with some probability, while negative values are influences that cause the negation of a node to occur with some probability. For instance, the arc between B and X has values -0.4 and 0.8. The first value, referred to as h, states that if B is true, then this will cause X to be false with probability 0.4, while the second value, referred to as g, states that if B is false, then this will cause X to be true with probability 0.8. Both h and g can take values in the interval (-1, 1). All non-root nodes are assigned a baseline probability, which is similar to the “leak” probability in the Noisy-Or approach. This probability is the user-assigned assessment that the event would occur independently of the modeled influences in a net.

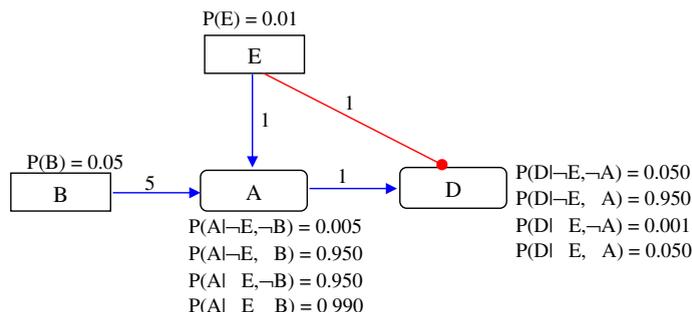


Fig. 1. A Sample Influence Net.

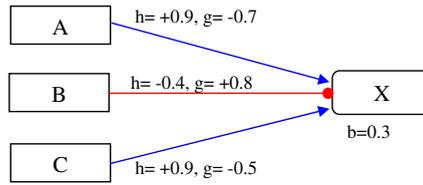


Fig. 2. An Influence Network with CAST logic parameters.

There are four major steps in the CAST logic algorithm that converts the user-defined parameters into conditional probabilities:

- (a) Aggregate positive causal strengths.
- (b) Aggregate negative causal strengths.
- (c) Combine the positive and negative causal strengths.
- (d) Derive conditional probabilities.

In Fig. 2, there are eight conditional probabilities that need to be computed to obtain the marginal probability of  $X$ . Mathematically, the marginal probability of  $X$  is computed as

$$\begin{aligned}
 P(X) = & P(X | \neg A, \neg B, \neg C)P(\neg A, \neg B, \neg C) + P(X | \neg A, \neg B, C)P(\neg A, \neg B, C) + P(X | \neg A, B, \neg C)P(\neg A, B, \neg C) \\
 & + P(X | \neg A, B, C)P(\neg A, B, C) + P(X | A, \neg B, \neg C)P(A, \neg B, \neg C) + P(X | A, \neg B, C)P(A, \neg B, C) \\
 & + P(X | A, B, \neg C)P(A, B, \neg C) + P(X | A, B, C)P(A, B, C)
 \end{aligned} \quad (1)$$

The four steps, described above, are used to calculate each of these eight conditional probabilities. For instance, to calculate the probability  $P(X|A,B,\neg C)$ , the  $h$  values on the arcs connecting  $A$  and  $B$  to  $X$  and the  $g$  value on the arc connecting  $C$  to  $X$  are considered. Hence, the set of causal strengths is  $\{0.9, -0.4, -0.5\}$ .

### 2.1.1. Aggregate the positive causal strengths

In this step, the set of causal strengths with positive influence are combined. They are aggregated using the equation

$$PI = 1 - \prod_i (1 - C_i) \quad \forall C_i > 0$$

where  $C_i$  is the corresponding  $g$  or  $h$  value having positive influence and  $PI$  is the combined positive causal strength. For our example

$$PI = 1 - (1 - 0.9) = 0.9$$

### 2.1.2. Aggregate the negative causal strengths

In this step, the causal strengths with negative values are combined. The equation used for aggregation is

$$NI = 1 - \prod_i (1 - C_i) \quad \forall C_i < 0$$

where  $C_i$  is the corresponding  $g$  or  $h$  value having negative influence and  $NI$  is the combined negative causal strength. Using the above equation, the aggregate negative influence is found to be:

$$NI = 1 - (1 - 0.4)(1 - 0.5) = 0.7$$

### 2.1.3. Combine positive and negative causal strengths

In this step, aggregated positive and negative influences are combined to obtain an overall net influence. The difference of these aggregated influences is taken. The overall influence is obtained by taking the ratio of this difference and the corresponding promoting or inhibiting influence. Mathematically,

$$\begin{aligned}
 & \text{If } PI > NI \\
 AI &= \frac{PI - NI}{1 - NI} \\
 & \text{If } NI > PI \\
 AI &= \frac{NI - PI}{1 - PI}
 \end{aligned}$$

Thus, the overall influence for the current example is

$$AI = (0.9 - 0.7)/(1 - 0.7) = .66$$

#### 2.1.4. Derive conditional probabilities

In the final step, the overall influence is used to compute the conditional probability value of a child for the given combination of parents.

$$\begin{aligned} P(\text{child} | j\text{th state of parent states}) &= \text{baseline} + (1 - \text{baseline}) * AI && \text{when } PI \geq NI \\ &= \text{baseline} - \text{baseline} * AI && \text{when } PI < NI \end{aligned}$$

Using the above equation,  $P(X|A,B,-C)$  is obtained as

$$P(X | A, B, -C) = 0.5 + 0.5 * 0.66 = .863$$

The steps explained above are repeated for the remaining seven conditional probabilities in Eq. (1). It should be noted that, if the experts had sufficient time and knowledge of the influences, then the conditional probability table for each node can be used instead of  $g$  and  $h$  values. Furthermore, after estimating the conditional probability table, if some entries do not satisfy the expert, then those entries can be modified and then used for computing the marginal probability of a node.

### 3. Timed Influence Nets

Influence Nets were designed to capture *static* interdependencies among variables in a system. However, a situation where the impact of a variable takes some time to reach the affected variable(s) cannot be captured by an IN. Wagenhals et al. [32] have added a special set of temporal constructs to the basic formalism of Influence Nets. The temporal constructs allow a system modeler to specify delays associated with nodes and arcs. These delays may represent the information processing and communication delays present in a given situation. For example, in Fig. 1, the inscription associated with each arc shows the corresponding time delay it takes for a parent node to influence a child node. For instance, event B influences the occurrence of event A in 5 time units.

The purpose of building a TIN is to evaluate and compare the performance of alternative courses of action. The impact of a selected course of action on the desired effect is analyzed with the help of a *probability profile*. Consider the net shown in Fig. 1. Suppose it is decided that actions B and E are taken at time 1 and 7, respectively. Because of the propagation delay associated with each arc, the influences of these actions impact event D over a period of time. As a result, the probability of D changes at a different time instants. A probability profile draws these probabilities against the corresponding time line. The probability profile of event D is shown in Fig. 3. The following items characterize a TIN:

1. A set of random variables that makes up the nodes of a TIN. All the variables in the TIN have binary states.
2. A set of directed links that connect pairs of nodes.
3. Each link has associated with it a pair of parameters that shows the causal strength of the link (usually denoted as  $g$  and  $h$  values).
4. Each non-root node has an associated baseline probability, while a prior probability is associated with each root node.
5. Each link has a corresponding delay  $d$  (where  $d \geq 0$ ) that represents the communication delay.
6. Each node has a corresponding delay  $e$  (where  $e \geq 0$ ) that represents the information processing delay.

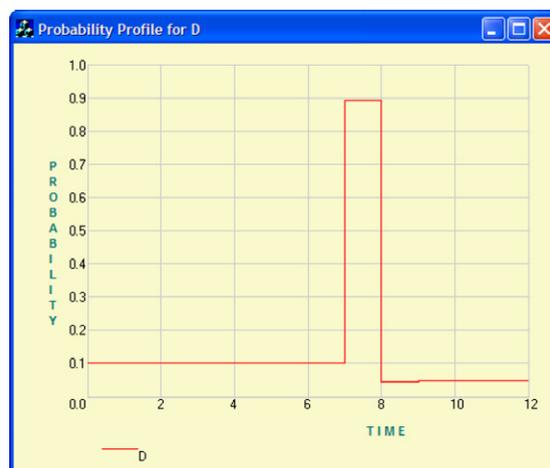


Fig. 3. Probability profiles of event D.

7. A pair  $(p, t)$  for each root node, where  $p$  is a list of real numbers representing probability values. For each probability value, a corresponding time interval is defined in  $t$ . In general,  $(p, t)$  is defined as

$$([p_1, p_2, \dots, p_n], [[t_{11}, t_{12}], [t_{21}, t_{22}], \dots, [t_{n1}, t_{n2}]])$$

where  $t_{i1} < t_{i2}$  and  $t_{ij} > 0 \forall i = 1, 2, \dots, n$  and  $j = 1, 2$ .

The last item in the above list is referred to as input scenario, or sometimes (informally) as course of action. Formally, a TIN is described by the following definition.

**Definition 2.** Timed Influence Net (TIN)

A *Timed Influence Net* is a tuple  $(\mathbf{V}, \mathbf{E}, \mathbf{C}, \mathbf{B}, \mathbf{D}_E, \mathbf{D}_V, \mathbf{A})$  where

$\mathbf{V}$ : set of Nodes,

$\mathbf{E}$ : set of Edges,

$\mathbf{C}$  represents causal strengths:

$$\mathbf{E} \rightarrow \{(\mathbf{h}, \mathbf{g}) \text{ such that } -1 < \mathbf{h}, \mathbf{g} < 1\},$$

$\mathbf{B}$  represents Baseline/Prior probability:  $\mathbf{V} \rightarrow [0, 1]$ ,

$\mathbf{D}_E$  represents Delays on Edges:  $\mathbf{E} \rightarrow \mathbf{Z}^+$ , (where  $\mathbf{Z}^+$  represent the set of positive integers)

$\mathbf{D}_V$  represents Delays on Nodes:  $\mathbf{V} \rightarrow \mathbf{Z}^+$ , and

$\mathbf{A}$  (input scenario) represents the probabilities associated with the state of actions and the time associated with them.

$\mathbf{A}: \mathbf{R} \rightarrow \{([p_1, p_2, \dots, p_n], [[t_{11}, t_{12}], [t_{21}, t_{22}], \dots, [t_{n1}, t_{n2}]]) \text{ such that } p_i \in [0, 1], t_{ij} \in \mathbf{Z}^+ \text{ and } t_{i1} \leq t_{i2}, \forall i = 1, 2, \dots, n \text{ and } j = 1, 2 \text{ where } \mathbf{R} \subset \mathbf{V}\}$  (where  $\mathbf{Z}^+$  represent the set of nonzero positive integers)

3.1. Transformation of Timed Influence Nets into Dynamic Bayesian Network

Haider and Zaidi [12] have developed an algorithm that transforms a TIN into a Time Sliced Bayesian Network (commonly known as Dynamic Bayesian Network). The transformation algorithm was developed with the aim of combining the advantages of both paradigms. A TIN provides a compact and intuitive way of modeling time-dependent uncertain domains, and thus can be used as a front-end tool for model building and course of action evaluation. The conversion to a DBN, on the other hand, makes it possible to use a variety of analysis and belief updating algorithms developed for DBNs. The algorithm is briefly explained with the help of TIN in Fig. 4 and the corresponding DBN in Fig. 5. Readers interested in the detailed process should refer to [12]. There are four major steps in the transformation algorithms:

1. Let  $M$  be the maximum path length between the root nodes and target nodes. For the TIN in Fig. 4,  $M$  is 5, which is the path length of the path A–C–D.

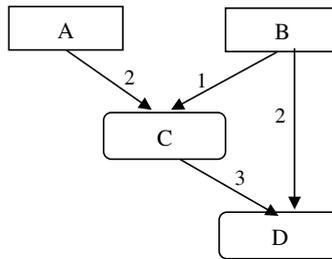


Fig. 4. A sample Timed Influence Net.

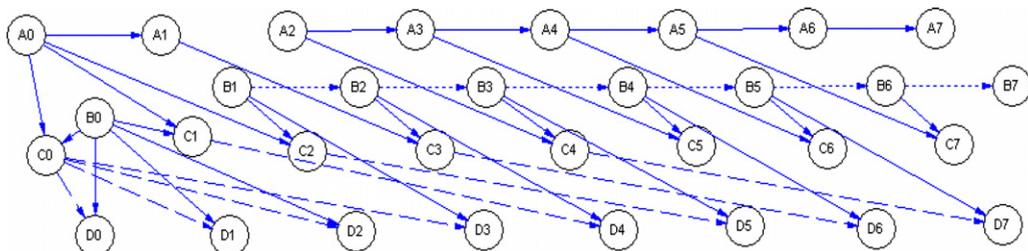


Fig. 5. A DBN obtained through the transformation algorithm.

2. Let  $S$  be the maximum time stamp associated with the root nodes as provided by the input scenario. Suppose in the given example, action  $A$  is taken at time 2 while action  $B$  is taken at time 1. Thus,  $S$  is set to 2.
3. Draw  $M + S$  time slices in the resultant DBN. The nodes are connected according to the time delays on the arcs. For instance, the arc delay between  $B$  and  $D$  is 2, thus  $D_5$  is connected to  $B_3$ ,  $D_4$  is connected to  $B_2$ , and so on. The subsequent indices of a root node (representing actionable events) are also connected except for the time when an action is taken.
4. The conditional probability tables are filled with the help of the CAST logic parameters, as discussed in Section 2.1.

#### 4. Enhancements to TIN

Despite their ability to represent complex situation in a compact way, TINs lack the ability to model certain kind of temporal relationship. For instance, there are situations when the orders in which actions are executed play a very important role in achieving a desired effect. Two different sequences of actions may have very contrasting impacts on the desired effect. This phenomenon happens when the probability of an event at a particular time instants depends upon its probability in the previous time instants. Currently, TINs fail to capture this phenomenon. Thus, they do not capture the impact of the probability of a node at previous time instant on its current probability. Furthermore, they do not remember the sequence in which actions take place. As a result of this *memoryless property*, no matter what the sequence of the actions is, the final probability of achieving a desired effect remains the same.

Another limitation of TINs is their inability to model *time-varying* influences. They assume that the influence of a cause remains the same throughout a campaign. This assumption may prove to be unrealistic in many situations. In reality, events happen and they influence other relevant events. There are situations, however, when the intensity of an influence decays over time. Thus, an event having a very strong influence at the time of its occurrence on another event might have an insignificant influence after a certain period of time. For example, a resolution passed by the United Nations has a very strong impact on the concerned parties at the time of its approval. As the time passes, the resolution starts losing its affect and after some period of time it completely loses its importance unless the problem is solved or it is backed up by another resolution on the same subject.

The following sub-sections suggest structural and parametric enhancements to the TIN-based modeling approach that overcome its above limitations. The enhancements allow a system modeler to specify (a) time-varying influences and (b) dependence of the current states of an event on its previous states. Together these enhancements make it possible to model the impact of repetitive actions in a dynamic situation. TINs with the proposed added constructs are termed as Dynamic Influence Nets (DIN).

##### 4.1. Modeling of memory

The existing TINs are not capable of modeling the impact of different sequences of actions on a desired effect. This behavior is because of the underlying assumption in TINs that events are *memoryless*, i.e. the probability of occurrence of an event at a particular time instant does not depend upon its own probabilities of occurrence during the previous time instants. As a consequence, the probability of an event depends only upon the actions executed so far and not on the sequence in which these actions are executed. The presented approach adds an optional *self-loop* to each node. The events having self-loops are no longer assumed to be memoryless. Like other arcs in a TIN, a self-loop is also specified using the CAST logic. A higher value (either positive or negative) of the parameters imitates strong memory while a lower value imitates weak memory. If both parameters ( $h$  and  $g$ ) are set to zero then this is equivalent of having no self-loop. Thus, this class of TINs is a superset of the TINs that were defined in Definition 2. In the TIN in Fig. 1, if events  $A$  and  $D$  depend upon their previous states, then this phenomenon is captured by adding a self-loop to each of them as shown in Fig. 6.

The addition of self-loop not only changes the final probability of the variable of interest, but it also has an affect on the trajectory of the probability profile. Consider the TIN shown in Fig. 7. It has three variables  $A$ ,  $B$ , and  $C$ . In the absence of a self-loop, the probability of event  $C$  depends only upon the probability of its parents, that is,  $A$  and  $B$ . Suppose two courses of action are required to be evaluated for this model. In the first course of action (COA 1), actions  $A$  and  $B$  are taken at times 10 and 12, respectively while in the second course of action (COA 2),  $A$  and  $B$  are taken at time 12 and 10, respectively. The respective probability profiles of  $C$  as a result of these courses of action are shown in Figs. 8a and 6b. Despite the fact that

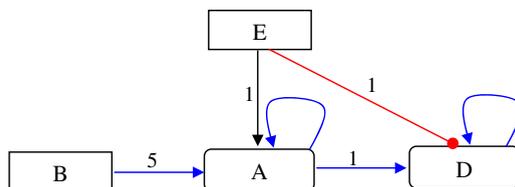


Fig. 6. A Timed Influence Net with self-loop.

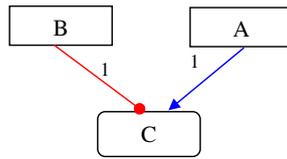
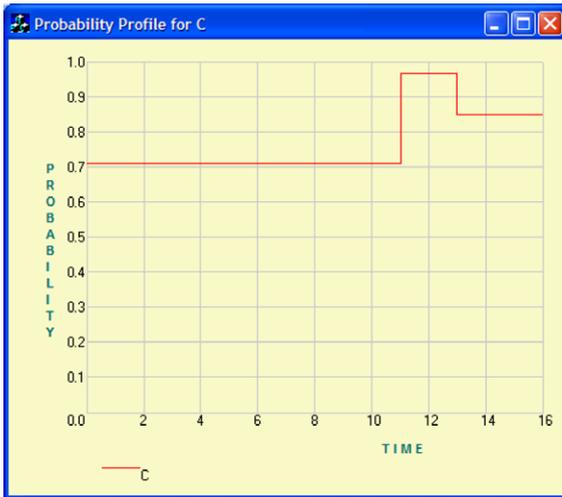
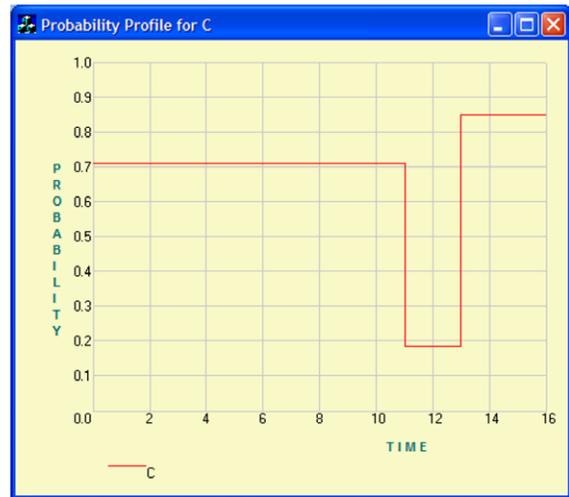


Fig. 7. A TIN having 3 nodes.



(a) COA 1



(b) COA 2

Fig. 8. Probability profiles of event C in the TIN in Fig. 7.

the trajectories shown in the two profiles differ significantly, the final probability of event C is same (0.85) in both profiles. This behavior is due to the fact that the underlying TIN model is memoryless. Thus, no matter what the sequence of actions A and B is, the likelihood of occurrence of C is same once both actions are taken.

In contrast to the given situation, suppose the likelihood of C at a particular time instance depends upon its own likelihood in the past. The proposed methodology attempts to model this situation by adding a self-loop to event C. The modified TIN is shown in Fig. 9. The text associated with the self-loop shows the corresponding CAST logic parameters. In addition to their normal semantics, the parameters attached to a self-loop also represent the strength of the memory associated with the corresponding variable. For instance, high values of g and h strongly cause a node to remain in its previous state, while lower values of g and h represent a weak memory and thus the previous state of a variable does not have a big influence on its current state. The two courses of action described earlier (COA 1 and COA 2) are executed for the model in Fig. 9a and the respective profiles are shown in Fig. 10.

It can be seen from the profiles that the final probability of event C is different in the two profiles. This change in the behavior of the TIN occurs because of the fact that now the present likelihood of C depends upon its likelihood in the past along with the probabilities of its parents. For instance, in the profile in Fig. 10a, event A happens first which causes an increase in the probability of C (0.85) as the occurrence of A has a strong positive influence on the occurrence of C. B happens after A. Despite its negative influence, B fails to decrease the likelihood of C as C has a strong memory that causes it to remain in the previous state along with the fact that a strong positive influence from A counterbalances a moderate negative influ-

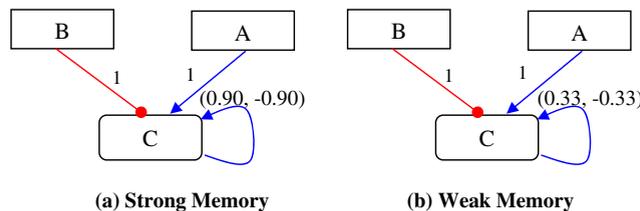


Fig. 9. Different levels of memory modeled using self-loops.

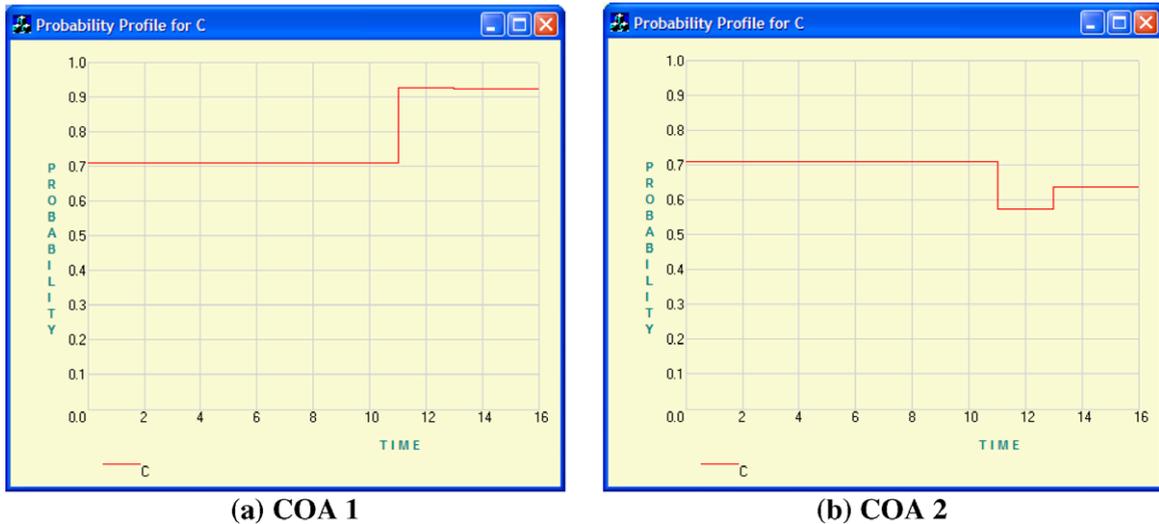


Fig. 10. Probability profiles of event C in the TIN of Fig. 7a.

ence from B. Thus, the final probability of C is 0.87. In the second profile (Fig. 10b), B happens first and due to its negative influence on C the probability of C is decreased to 0.56. A happens next and it slightly increases the probability of C to 0.63 but not as much as it is increased in COA 1 because of the dependency of C on its previous state. While comparing the profiles in Figs. 6 and 8, it can be noticed that the profiles have quite a different behavior in both courses of action.

If the  $h$  and  $g$  values associated with a self-loop are low, then the loop represents a weak influence of the previous state of a node on its current state. Suppose in the model in Fig. 9a, the  $g$  and  $h$  values associated with the self-loop are revised and are as shown in Fig. 9b. The same two courses of action (COA 1 and COA 2) are executed in this situation and the resultant probability profiles are shown in Fig. 11: a weak memory has resulted in the final probabilities very close to what is obtained in the profiles based on a memoryless TIN (Fig. 8).

Up until now, it is assumed that a node's likelihood at a previous time stamp is used to update its current likelihood when a new piece of information arrives from one of its parents. A self-loop can also be used to update the probability of a node at a regular time interval. This time interval is specified as the delay associated with a self-loop. Thus a self-loop can be used to model *decay* in the belief of a node as the time passes and no new information from its parents influences it. Suppose in the model in Fig. 9, the self-loop associated with node C has a delay of 1 time unit which means that the probability of C is updated after every 1 time unit regardless of whether there is new information coming from its parents or not. In the remainder of this paper, if the delay associated with a self-loop has a value of zero then it means that a previous value of a node is used

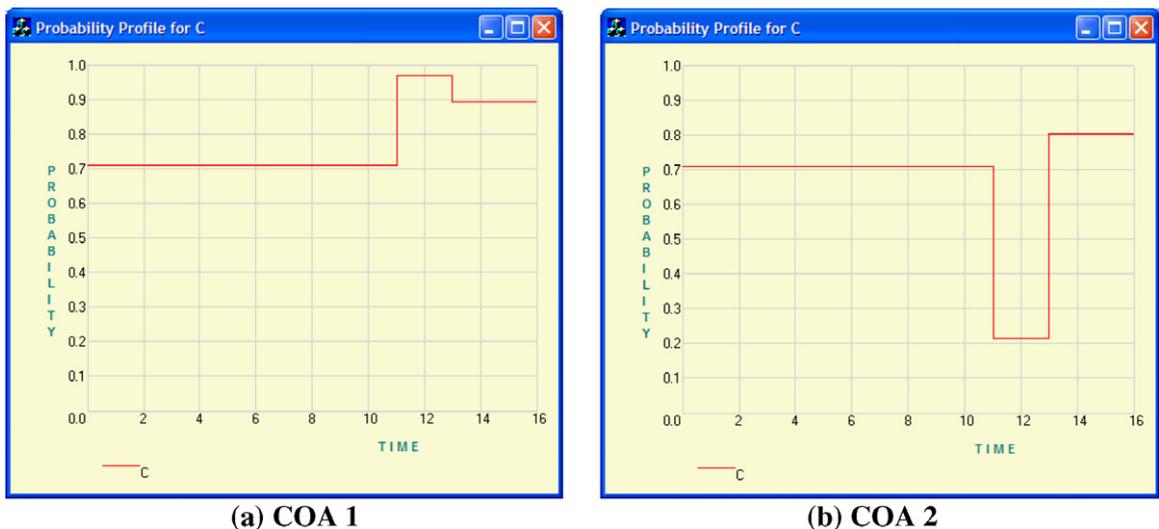


Fig. 11. Probability profiles of event C in the TIN in Fig. 9b.

to update its current likelihood only when there is new information coming from its parents. Positive values other than zero indicate that the update would occur at a regular time interval.

4.2. Modeling of time-varying influences

Events happen and they influence other relevant events. In many cases, the intensity of their influences decays over time. Thus, an event having a very strong influence at the time of its occurrence on another event might have an insignificant influence after a certain period of time. In other words, the influence of an event is *time-variant*. The *time-varying* property also holds true for the state of an action. An action may occur in two different states during two different time intervals. In TINs terminology, these two types of time-varying properties are referred to as *persistence*. The one related to the time-dependent influence of an action is called *persistence of influence*, while the one related to the time-dependent state of an action is called *persistence of action*. Among these two types of persistence, a TIN currently models the latter one only. It assumes that the causal strength of the influences does not change over time, i.e., the underlying stochastic model is *stationary*. Thus, it lacks the ability to model persistence of influence. This paper attempts to overcome this limitation of TINs. The presented approach enables a system modeler to model *non-stationary* influences. Instead of asking a modeler to specify single-valued influences, the approach would allow the modeler to specify various strengths of influences and their corresponding window of effectiveness.

Consider the TIN in Fig. 12. The prior probability of nodes A and B at time 0 is 0.05 and 0.1, respectively. Action A is taken at time 4 while the probability of occurrence of B becomes 0.6 at time 7 and 1 at time 10. The qualitative time-varying influences are also shown in Fig. 12. The actual CAST logic parameters corresponding to these qualitative statements are shown in Figs. 11, 12. The time-varying influences are read in the following manner: A has a high positive influence on C, if the change occurred at A is 2 to 3 time units old, and its influence is moderate, if the change occurred at A is 4 to 5 time units, while its influence is low if the change occurred at A is more than 6 time units old. For simplicity “strong influence” is assumed to mean that both h and g have the same values though with opposite signs (one is positive and the other is negative). Similarly, B has a strong negative influence on C when the change that occurred at B is 1 to 2 time units old, while it has a low influence when the change occurred at B is more than 2 time units old. Due to the provided input scenario, the probability of C is updated at time 6, 8, and 11 as the time delays between A and C and B and C are 2 and 1, respectively. C is updated at time 6 because action A is taken at time stamp 4. The last change that occurred at B is at time 0. Thus, the probability of B used in computing the marginal probability of C is 0.1. Since this value is 6 time units old, a low negative influence of B on C is considered while computing the CPT values for node C. The TIN with this particular instance of the CAST logic parameters along with the prior probabilities is shown in Fig. 13. The Conditional Probability Table (CPT) values computed under this situation are also shown beside node C. Based on the parameters shown in the figure, the marginal probability of C at time 6 is computed as given below.

$$P(C) = P(C | \neg A, \neg B) \times P(\neg A) \times P(\neg B) + P(C | \neg A, B) \times P(A) \times P(B) + P(C | A, \neg B) \times P(\neg A) \times P(B) + P(C | A, B) \times P(A) \times P(B) = 0.93$$

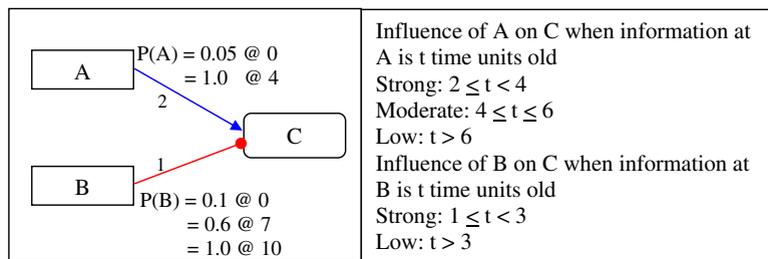


Fig. 12. A TIN having time-variant influences.

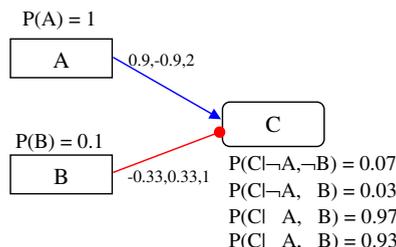


Fig. 13. An instance of the TIN in Fig. 12.

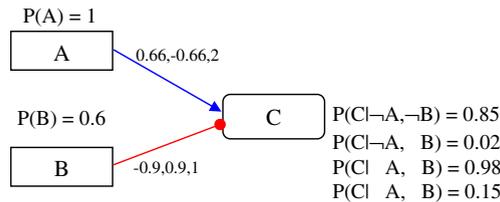


Fig. 14. Another instance of the TIN in Fig. 12.

Table 1  
Non-stationary CPTs

Parents combination	6	Time	
		8	11
P(C -A, -B)	0.07	0.85	0.93
P(C -A, B)	0.03	0.02	0.03
P(C A, -B)	0.97	0.98	0.97
P(C A, B)	0.93	0.15	0.07

The next update of P(C) occurs at time 8. At this time instance, the marginal probability of A is 4 time units old; thus a moderate positive influence of A on C is considered while computing the CPT values. The probability of B is only 2 time units old and has a strong negative influence on C. The resultant parameters, along with the CPT values, are shown in Fig. 14. The probability of C at time 8 is computed as shown below.

$$P(C) = P(C | -A, -B) \times P(-A) \times P(-B) + P(C | -A, B) \times P(A) \times P(B) + P(C | A, -B) \times P(-A) \times P(B) + P(C | A, B) \times P(A) \times P(B) = 0.48$$

The last update of P(C) occurs at time 11. The marginal probability of A is 7 time units old, while B's is 2 time units old. Thus, a low positive influence from A and a strong negative influence from B are considered. The updated probability of C is found to be 0.07. The above analysis demonstrated how non-stationary CAST logic parameters have resulted in non-stationary CPT values that are used in computing the probability of C at various time stamps. Thus, despite the fact that an action is still in effect, it may lose its significance as time passes by. The non-stationary CPT values used in the above computations are presented in Table 1 along with the time of their computation.

### 5. Dynamic Influence Nets

The incorporation of the suggested structural and parametric changes in TINs, as described in the previous sections, enables a system modeler to observe the impact of repeated actions. For instance, an air-strike on a bridge makes it inoperable for several days. The current implementation of TINs would assume that the influence of the air-strike remains the same throughout the campaign. It is obvious that the assumption is unrealistic. Furthermore, in the event of a new air-strike, a TIN would discard the impact of the previous air-strike as the events in a TIN are assumed to be memory-less. The proposed approach, which allows time-varying influences and incorporation of memory through self-loops, models this situation in a more intuitive manner. Like other arcs in a TIN, a self-loop also represents influence – from the previous state of a node to its current state. Thus, time-varying parameters can be associated with a self-loop too. For the air-strike example, the presence of self-loop would combine the influences of both (or many) air-strikes while the strength of the self-loop accounts for the time delay between the two air-strikes. If the timing of two air-strikes is far apart, then there is almost no influence of the first strike on the operability of the bridge (provided that the bridge has been rebuilt), but if the two strikes occur very close in time then their impact would be more destructive. In other words the impact of two actions on the effect convolves. The issue is further explained with the help of the following example. Suppose the variables in the model in Fig. 12 have the following descriptions:

- A – Regional Countries Opposes Sanctions against Country R.
- B – Country G Threatens to Take Unilateral Actions against Country R .
- C – Leader of Country R Decides to Accept UN Demands.

Further assume that the belief of event C at a particular time depends upon its own belief at a previous time instance though not very strongly. This behavior is modeled by adding a self-loop having a low influence on event C. The resultant model is shown in Fig. 15. The probabilities of actionable events A and B are changed at various time stamps, as described earlier and shown in the figure. The resultant probability profile of C is shown in Fig. 16a. If the same situation is modeled using an existing TIN, that is, without the self-loop and time-invariants CAST logic parameters, then the resultant profile of C

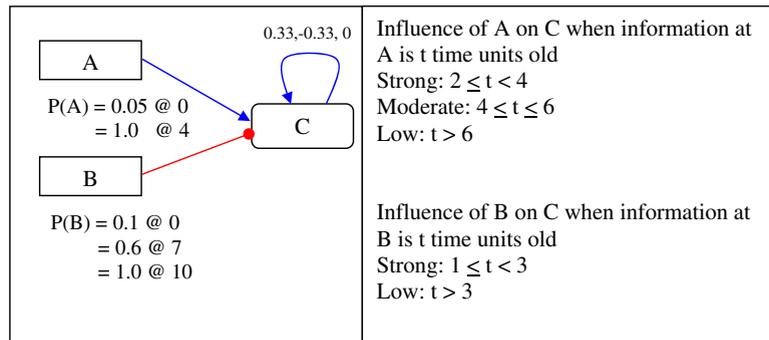


Fig. 15. A TIN with self-loop and time-varying influences.

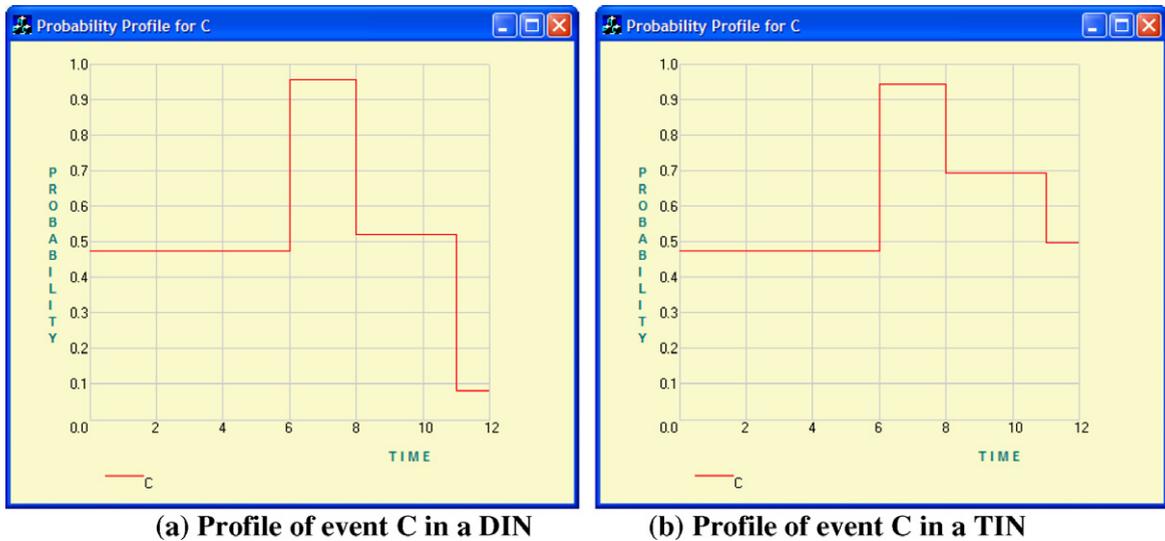


Fig. 16. Comparison of profiles generated by a TIN and a DIN.

is as shown in Fig. 16b. Currently, there is no validation technique that helps us in identifying which profile is a better representation of the situation at hand, but it can be said that the profile shown in Fig. 16a is more in agreement with intuition than the profile of Fig. 16b. The impact of B is more dominating in the profile of Fig. 16a as event A happened 6 time units earlier and has lost its significance. Furthermore, the previous state of event C also has an impact on its current state. Thus, a different sequence of actions would have resulted in a completely different outcome. Profile in Fig. 16b fails to capture these characteristics.

The applicability of the above concepts is not just limited to the toy examples discussed so far. Fig. 17 shows a portion of a Timed Influence Net developed to model the political crisis that occurred in East Timor during the final years of the previous decade. Due to space limitations only a small portion of the model is shown in the figure. Readers interested in a detail study of the model and the corresponding analysis should refer to [33]. The model was developed as a prototype for the Decision Support System for Coalition Operations developed by SPAWAR Systems Center – San Diego to support the Operations Planning Team of the Commander in Chief, U.S. Pacific Command. A closer look at many of the actionable events, such as announcements of US president and UN security general, reveals that these events have time-varying influences. Same is true for the actions taken by Indonesian government. Furthermore, the state of many events, such as rebels trust in coalition impartiality and their belief in their own strength to inflict heavy casualties, depend on their previous state. The incapability of TINs to model time-varying influences and self-loop resulted in modeling such situations with the assumption of time-invariant influences and sequences having no impact on an event's state. The constructs presented in this paper relaxes such modeling constraints and enhances the modeling power of TINs.

A similar case can be made about all the other realistic TINs reported in the literature. For instance, Wagenhals et al. [28] developed a TIN to assess a nation's ability to wage war based on damage effects to its civil infrastructure. Wentz and Wagenhals [35] developed a TIN to model broad-front national level actions needed to achieve an outcome that deterred a terrorist field cell from attacking. DeGregario et al. [6] developed a TIN to model certain aspects of the first Gulf war.

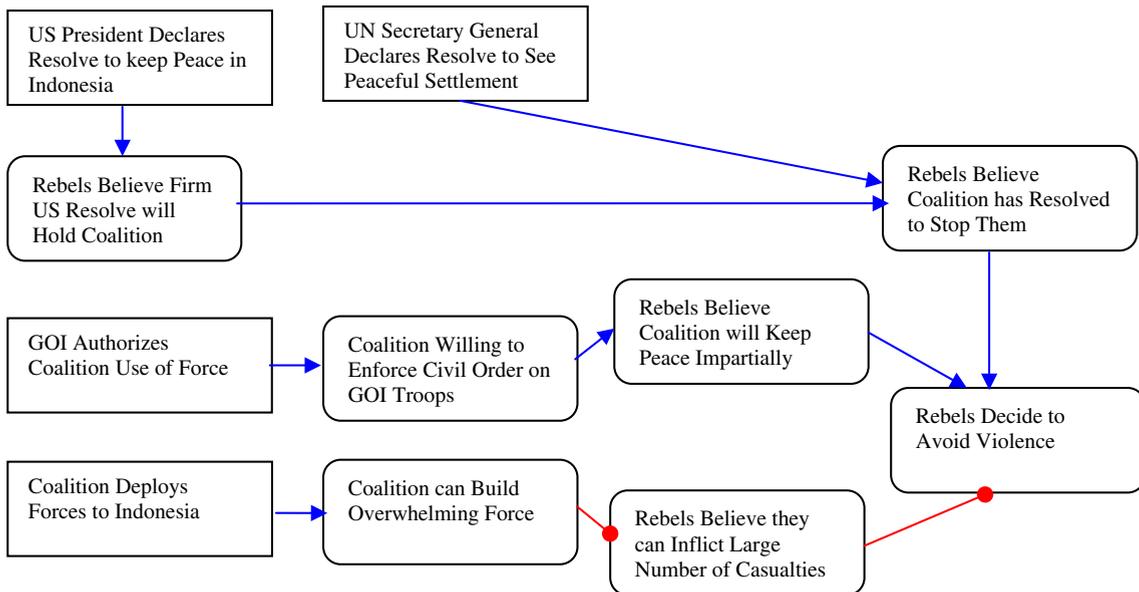


Fig. 17. A reduced version of the east Timor model.

Wagenhals and Wentz [34] developed a TIN to model chemical and biological threat from terrorists. Wagenhals and Levis [31] developed a TIN to evaluate a complex situation in which an adversary is embedded in a society from which it is receiving support. All these TINs were developed with the aim of capturing real world situations. But the incapability of TIN to capture time-varying influences and impact of different sequences of actions put limitations on the modeling process. The incorporation of the new constructs (self-loop and time-varying parameters) in TIN-based modeling and reasoning framework enhances the capabilities of this modeling paradigm in terms of capturing dynamic uncertain situations.

A TIN with these additional constructs has been defined as a Dynamic Influence Net (DIN). The following items characterize a DIN while a formal definition is given in Definition 3.

1. The nodes of a DIN are set of random variables. All the variables in the DIN have binary states.
2. A set of directed links that connect pairs of nodes. A node can also have an optional self-loop.
3. A pair  $(c, t)$  for each link, where  $c$  is a list of tuples representing the CAST logic parameters. For each element in  $c$ , a corresponding time interval is defined in  $t$ . This interval represents the time during which the corresponding element in  $c$  is in effect. In general,  $(c, t)$  is defined as

$$(((h_1, g_1)(h_2, g_2), \dots, (h_n, g_n)), [(t_{11}, t_{12}), (t_{21}, t_{22}), \dots, (t_{n1}, t_{n2})])$$

where  $t_{i1} < t_{i2}$  and  $t_{ij} > 0 \forall i = 1, 2, \dots, n$  and  $j = 1, 2$

4. Each non-root node has an associated baseline probability, while a prior probability is associated with each root node.
5. Each link has a corresponding delay  $d$  (where  $d \geq 0$ ) that represents the communication delay.
6. Each node has a corresponding delay  $e$  (where  $e \geq 0$ ) that represents the information processing delay.
7. A pair  $(p, t)$  for each root node, where  $p$  is a list of real numbers representing probability values. For each probability value, a corresponding time interval is defined in  $t$ . In general,  $(p, t)$  is defined as

$$([p_1, p_2, \dots, p_n], [(t_{11}, t_{12}), [t_{21}, t_{22}), \dots, [t_{n1}, t_{n2})])$$

where  $t_{i1} < t_{i2}$  and  $t_{ij} > 0 \forall i = 1, 2, \dots, n$  and  $j = 1, 2$

**Definition 3**

A Dynamic Influence Net is a tuple  $(V, E, C, B, D_E, D_V, A)$  where

**V**: set of Nodes,

**E**: set of Edges,

**C** represents causal strengths:

$$E \rightarrow \{((h_1, g_1)(h_2, g_2), \dots, (h_n, g_n)), [(t_{11}, t_{12}), (t_{21}, t_{22}), \dots, (t_{n1}, t_{n2})]) \text{ such that } -1 < h_i, g_i < 1, t_{ij} \rightarrow \mathbf{Z}^+ \text{ and } t_{i1} \leq t_{i2}, \forall i = 1, 2, \dots, n \text{ and } j = 1, 2\}$$

**B** represents Baseline/Prior probability:  $V \rightarrow [0,1]$ ,

$D_E$  represents Delays on Edges:  $E \rightarrow Z^+$ ,

$D_V$  represents Delays on Nodes:  $V \rightarrow Z^+$ , and

$A$  (input scenario) represents the probabilities associated with the set of actions and the time associated with them.

$A: R \rightarrow \{([p_1, p_2, \dots, p_n], [[t_{11}, t_{12}], [t_{21}, t_{22}], \dots, [t_{n1}, t_{n2}]])\}$  such that  $p_i = [0, 1]$ ,  $t_{ij} \rightarrow Z^+$  and  $t_{i1} \leq t_{i2}$ ,  $\forall i = 1, 2, \dots, n$  and  $j = 1, 2$  where  $R \subset V$

### 5.1. Comparison with DBNs

As an extension of TINs, DINs provide a compact and intuitive knowledge elicitation framework to model dynamic uncertain domains. Unlike a TIN, a DIN also has the capability to model time-varying influences and the impact of different sequences of actions. Furthermore, in contrast to an exponential number of parameters required for the specification of a DBN (especially in the case of non-stationary conditional probabilities), a DIN requires only a linear number of parameters for model specification. The DIN framework, however, is mainly suitable for courses of action evaluation and has no comprehensive provision to incorporate evidence which arrive during the execution of a course of action. Several algorithms, on the other hand, have been developed for belief updating in a DBN. A natural extension of the current work is to enhance the transformation algorithm of Haider and Zaidi [12] (also briefly discussed in Section 3.1) that converts a TIN into a DBN. A forthcoming paper will discuss the enhanced transformation algorithm for converting a DIN into a DBN and the additional advantages obtained through this transformation.

## 6. Conclusions

The paper presented structural and parametric enhancements to Timed Influence Nets. The new class of models based on these enhancements is named Dynamic Influence Nets. The enhancements enable a system analyst to model time-varying influences and to capture the impact of different sequences of actions on a desired effect. The incorporation of self-loop adds memory to the existing memory-less TIN. The addition of both self-loop and time-varying influences enables the analyst to model impacts of repeated actions on an effect. Currently, in the event of repeated actions, a TIN only considers the latest impact on the effect while ignoring the previous attempts; a DIN, on the other hand, convolves the impact of repeated actions on the desired effect and, thus, further enhances the capabilities of Influence Nets-based modeling paradigm.

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