Probabilistic Reasoning

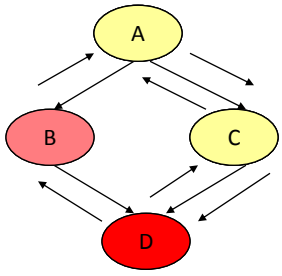
Unit # 10

Intractability of Inference in BN (Laskey)

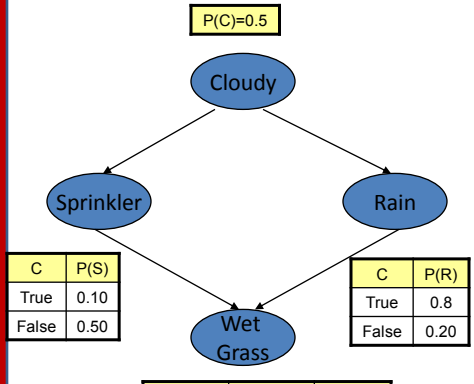
- **Belief propagation is NP-hard (Cooper, 1988)**
  - **NP-hard problems are problems which are at least as computationally complex as NP-complete problems**
  - **NP-complete is a class of decision problems that are computationally complex**
  - **There is a theorem which says that if there is an algorithm which solves one NP-Complete problem in polynomial time, then any NP-complete problem can be solved in polynomial time**
  - **No such algorithm has yet been found, and most people believe there is none**
- **Although the general problem appears to be very hard, tractable algorithms are available for certain types of networks**
  - **Singly connected networks**
  - **Some sparsely connected networks**
  - **Networks amenable to various approximation algorithms**

Inference In Multiply Connected Networks

- We get evidence on node D.
- As a result we need to update the marginal probabilities of the rest of the variables
- How to do it?



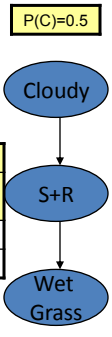
Exact Inference: Clustering



C	P(S)
True	0.10
False	0.50

C	P(R)
True	0.8
False	0.20

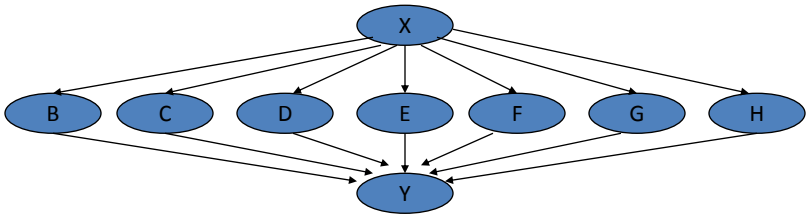
S	R	P(W)
True	True	0.99
True	False	0.90
False	True	0.90
False	False	0.00



	S+R			
C	True True	True False	False True	False False
True	0.08	0.02	0.72	0.18
False	0.10	0.40	0.10	0.40

S+R	P(W)
True True	0.99
True False	0.90
False True	0.90
False False	0.00

Problem with the Clustering Approach



- We are back to the same number of computations as was in the enumeration approach

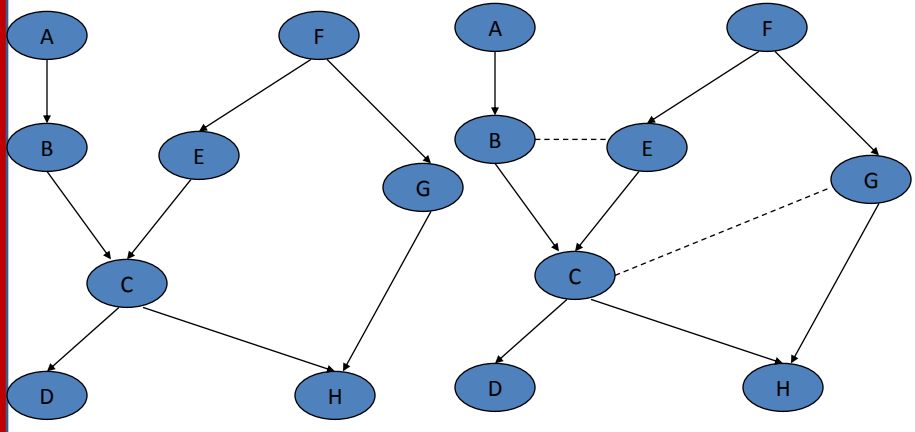
Overview of Junction/Clique Tree Algorithm (Laskey)

- The basic idea is to transform the graph into clusters of nodes so that the graph of clusters is singly connected and has the *junction tree property* (this property ensures that evidence propagates correctly)
- The junction tree becomes a permanent part of the knowledge representation, and changes only if the graph changes
- Constructing a junction tree from a Bayesian network is inherently non-modular, but the junction tree itself is a modular representation
- Beliefs are propagated in the junction tree using a local message-passing algorithm

How to Construct a JT

- Moralize the graph by adding links between parents of common children.
- Convert the given BN into an undirected graph by removing the arrowheads.
- Triangulate the graph.
- Order the nodes by maximum cardinality search.
- Find the cliques of the triangulated graph.
- Arrange as a junction tree.

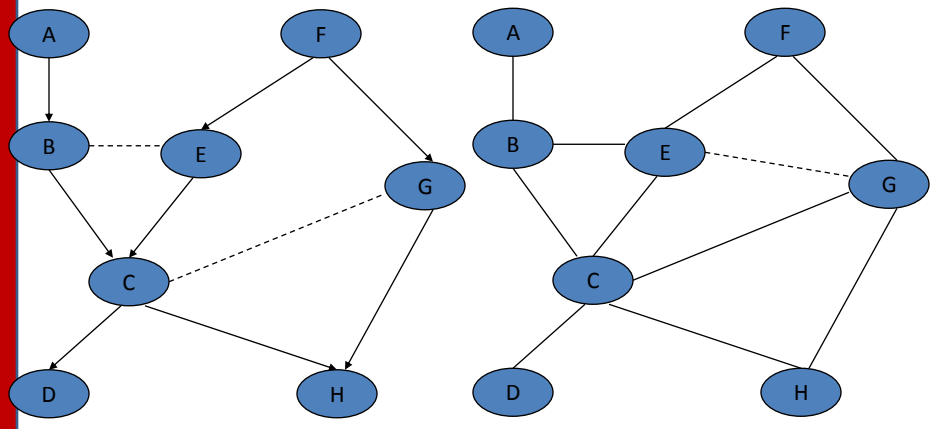
Example I (Laskey)



Example Bayesian Network

Moralize the graph

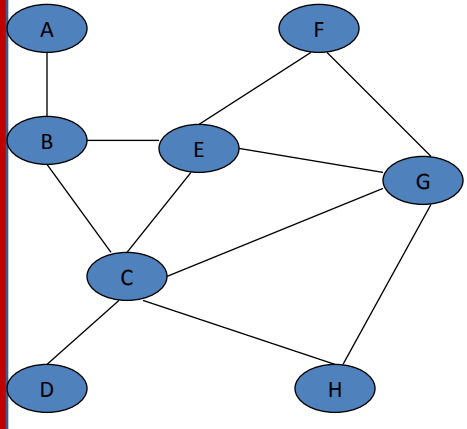
Example I (Cont'd)



Moralize the graph

Triangulate the graph

Example I (Cont'd)



Order Nodes in the Graph

Clique	Max. Vertex
AB	2
BEC	4
ECG	5
EGF	6
CGH	7
CD	8

Example I(Cont'd)

Clique	Max. Vertex
AB	2
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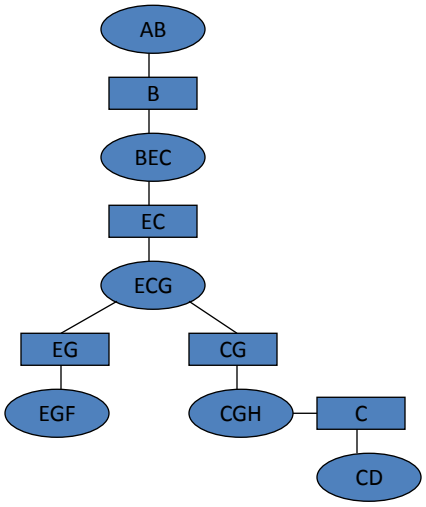


Properties of Clique Tree

- A clique tree is an undirected tree, where each node represents a set of variables, which is called a clique.
- A clique separator is the intersection between its two neighbouring cliques.
- Note that not every clique tree is a junction tree.
- A clique tree is a junction tree if and only if it has the *running intersection property or the junction tree property.*

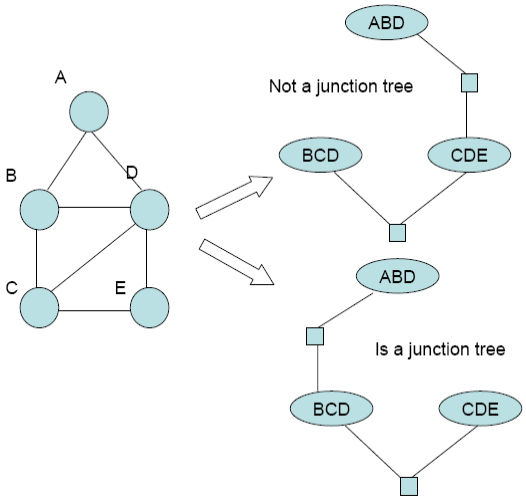
Properties of a Junction Tree (Zhang)

- The junction tree has the following characteristics:
  - it is an undirected tree, its nodes are clusters of variables
  - given two clusters, C_1 and C_2, every node on the path between them contains their intersection $C_1 \cap C_2$
  - a Separator, S, is associated with each edge and contains the variables in the intersection between neighbouring nodes

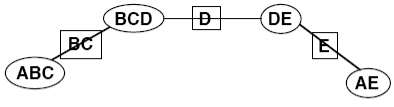
Junction Tree Property (Williams)

- A clique tree is a junction tree if it has the following junction tree property:
  - if a node appears in two cliques, it appears everywhere on the path between the cliques.
- *Mathematically, a clique tree possesses the junction tree property if for every pair of cliques V* and *W, all cliques on the (unique) path between V and W contain V ∩W.*
- For every triangulated graph there exists a clique tree which obeys the junction tree property
- Not all clique trees are junction trees

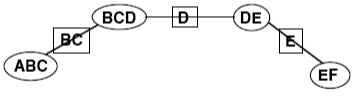
Junction Tree Examples (Jordan)



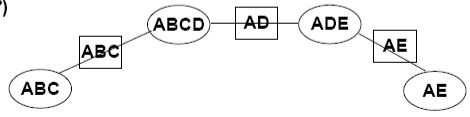
Junction Tree Examples (Laskey)



A cluster tree that is not a junction tree (*Why not?*)



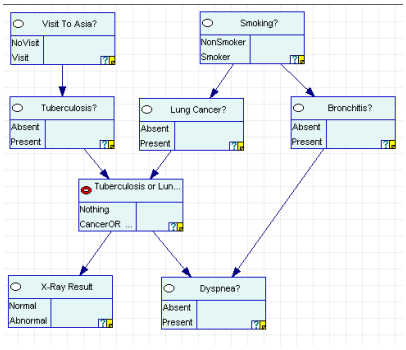
A junction tree (*Why?*)



Also a junction tree (*Why?*)

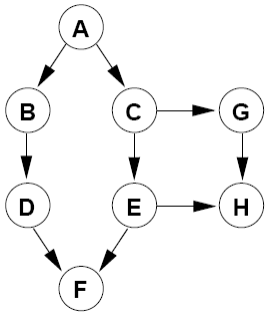
Example II

- Consider this BN and transform it into a junction tree by performing moralization, triangulation and maximum cardinality search.



Example III (Huang & Darwiche)

- Consider this BN and transform it into a junction tree by performing moralization, triangulation and maximum cardinality search.



Example IV

- Consider this BN and transform it into a junction tree by performing moralization, triangulation and maximum cardinality search.