Probabilistic Reasoning

Unit # 12

Likelihood Weighting (Laskey)

- Logic sampling can be very inefficient because it throws out so many observations.
- It can be modified to avoid throwing out observations.
- To take one random draw (one observation on all variables):
  - The evidence variables are fixed to their observed states.
  - Generate random samples for the non-evidence variables.
  - Weight the observation by a "sampling weight" that depends upon the evidence.
- Estimate P(Xt|Xe) as the weighted average.

Algorithm (From Jensen and Nielsen)

1. Let $(X_1, \ldots, X_n)$ be a topological ordering of the variables.
2. For $j = 1$ to N:
   a) w:=1.
   b) For $i = 1$ to n:
      - Let $\mathbf{x}'$ be the configuration of $(X_1, \ldots, X_{i-1})$ specified by $\mathbf{e}$ and the previous samples.
      - If $X_i \notin \mathcal{E}$, then:
         - Sample a state x_i for X_i using $P(X_i \mid \text{pa}(X_i) = \pi)$, where $\text{pa}(X_i) = \pi$ is consistent with $\mathbf{x}'$.
      - else
         $w := w \cdot P(X_i = e_i \mid \text{pa}(X_i) = \pi)$, where $\text{pa}(X_i) = \pi$ is consistent with $\mathbf{x}'$.
   c) $N(X_k = x_k) := N(X_k = x_k) + w$, where x_k is the sampled state for X_k.
3. Return:

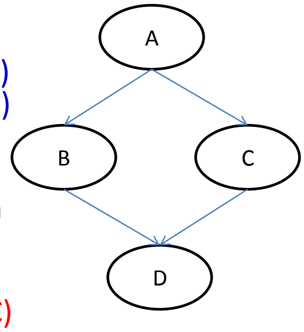
$$P(X_k = x_k \mid \mathbf{e}) \approx \frac{N(X_k = x_k)}{\sum_{x \in \text{sp}(X_k)} N(X_k = x)}.$$

Loopy Belief Propagation

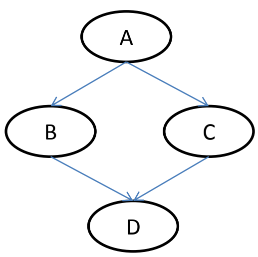
- Originally the message-passing belief propagation was developed for polytrees.
- In loopy belief propagation, we make certain independence assumption (assuming that no loop exists when one exist)

Concept Behind Loopy Belief Propagation

- $P(B) = P(B|A)P(A)+P(B|\sim A)P(\sim A)$
- $P(C) = P(C|A)P(A)+P(C|\sim A)P(\sim A)$
- $P(D) = P(D|B,C)P(B,C) + P(D|B,\sim C)P(B,\sim C) + P(D|\sim B,C)P(\sim B,C) + P(D|\sim B,\sim C)P(\sim B,\sim C)$
- If we assume that B and C are independent (when they are not as obvious from the structure) then we can compute the above equation as
- $P(D) = P(D|B,C)P(B)P(C) + P(D|B,\sim C)P(B)P(\sim C) + P(D|\sim B,C)P(\sim B)P(C) + P(D|\sim B,\sim C)P(\sim B)P\sim C)$



Examples



SET I

P(A) = 0.3
P(B|A)=0.7, P(B|~A)=0.05
P(C|A)=0.2, P(C|~A)=0.9
P(D|B,C) =0.9, P(D|B,~C) =0.6
P(D|~B,C)=0.8, P(D|~B,~C)=0.02

SET II

P(A) = 0.3
P(B|A)=0.9, P(B|~A)=0.01
P(C|A)=0.02, P(C|~A)=0.95
P(D|B,C) =0.99, P(D|B,~C) =0.95
P(D|~B,C)=0.90, P(D|~B,~C)=0.02

- Using Set I and II, compute the probability of D, P(D), using loopy belief propagation and compare the results with the ones obtained through GeNIe (exact, logic sampling and likelihood weighting). What do you observe?