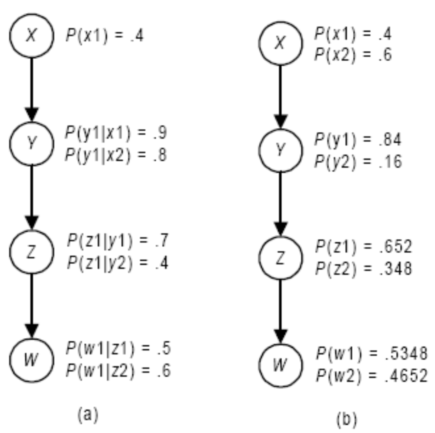
Probabilistic Reasoning

Unit # 4

Inference in Singly Connected Networks

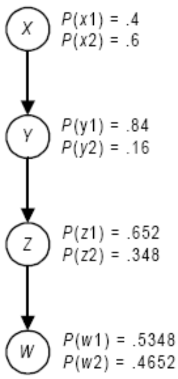


P(y1) = P(y1 | x1) P(x1)
+ P(y1 | x2) P(x2)
P(y2) = 1 – P(y1)

P(z1) = P(z1 | y1) P(y1)
+ P(z1 | y2) P(y2)
P(z2) = 1 – P(z1)

P(w1) = ???

Inference in Singly Connected Networks



Suppose we get evidence that w1 is true, i.e., P(w1) = 1.

Now compute the posterior probabilities:
P*(z1), P*(y1), P*(x1)

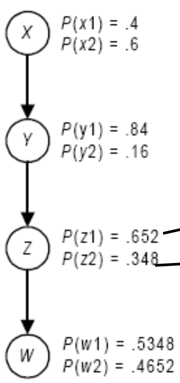
P*(z1) = P(z1 | w1) P(w1) ← P(w1)=1
+ P(z1 | w2) P(w2) ← P(w2)=0

Computing P(z1 | w1) using Bayes theorem:
P(z1 | w1) = P(w1 | z1) P(z1) / P(w1)
P(z1 | w1) = 0.5 x 0.652 / 0.5348 = 0.61

=>
P*(z1) = 0.61 * 1 + 0 = 0.61

Computation of P*(y1) on the next slide

Inference in Singly Connected Networks



Now update the probability of Y.
P*(y1) = P(y1 | z1) P(z1) ← P(z1)=0.61
+ P(y1 | z2) P(z2) ← P(z2)=0.39

} Computed on the previous slide

Computing P(y1 | z1) and P(y1 | z2) using Bayes theorem:
P(y1 | z1) = P(z1 | y1) P(y1) / P(z1)
P(y1 | z1) = 0.7 x 0.84 / 0.652 = 0.90

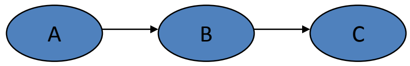
P(y1 | z2) = P(z2 | y1) P(y1) / P(z2)
P(y1 | z2) = 0.3 x 0.84 / 0.348 = 0.92

Finally plug the values in the equation at the top
P*(y1) = P(y1 | z1) P(z1) + P(y1 | z2) P(z2)
= 0.90 x 0.61 + 0.72 X 0.39
= 0.83
Confirm the results using Genie.

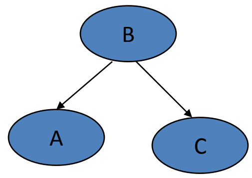
Finally compute P*(x1)

Conditional Dependence/Independence

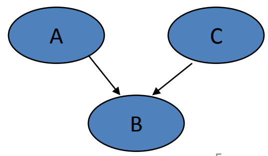
- **Causal Chains (Serial Connection)**
  - smoking causes cancer which causes dyspnoea
  - if we have evidence on B then A and C become independent



- **Common Causes (Diverging Connection)**
  - cancer is a common cause of the two symptoms, a positive XRay result and dyspnoea
  - If we have evidence on B then A and C become independent

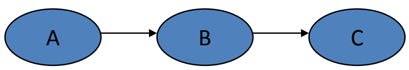


- **Common Effects (Converging Connection)**
  - Cancer is a common effect of pollution and smoking
  - If we have evidence on B then A and C become dependent



Serial Connection Example

- Consider the following BN



P(A) = 0.4

P(B|A) = 0.7
P(B|¬A) = 0.2

P(C|B) = 0.05
P(C|¬B) = 0.8

- Using the Bayesian Chain Rule, compute the joint distribution (i.e., eight values: P(A, B, C), P(A, B, ¬C), ………., P(¬A, ¬B, C) and P(¬A, ¬B, ¬C)
- Now compare
  - P(A | B, C) with P(A | B)
  - P(C | A, B) with P(C | B)

Practice Questions

- Develop a simple converging connection style BN (as shown in the earlier slide).
- After computing the joint probability distribution, verify that
  - P(A | B, C) = P(A | B)
  - P(C | A, B) = P(C | B)

- Develop a simple diverging connection style BN. After computing the joint distribution, verify that
  - P(A | C) = P(A) but P(A | B, C) != P(A | B)
  - P(C | A) = P(C) but P(C | A, B) != P(C | B)